PHYSICAL REVIEW E VOLUME 59, NUMBER 2 FEBRUARY 1999

Self-organization of traffic jams in cities: Effects of stochastic dynamics and signal periods

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(Received 9 October 1998)

We propose a cellular automata model for vehicular traffic in cities by combining (and appropriately modifying) ideas borrowed from the Biham-Middleton-Levine (BML) model of city traffic and the Nagel-Schreckenberg (NS) model of highway traffic. We demonstrate a phase transition from the ''free-flowing'' dynamical phase to the completely ''jammed'' phase at a vehicle density which depends on the time periods of the synchronized signals and the separation between them. The intrinsic stochasticity of the dynamics, which triggers the onset of jamming, is similar to that in the NS model, while the phenomenon of complete jamming through self-organization as well as the final jammed configurations are similar to those in the BML model. Using our model, we have made an investigation of the time dependence of the average speeds of the cars in the ''free-flowing'' phase as well as the dependence of flux and jamming on the time period of the signals.
[S1063-651X(99)51602-4]

PACS number(s): 05.40.−a, 05.60.−k, 89.40.+k

Over the last half century various concepts and techniques of fluid dynamics and statistical mechanics have been successfully applied to understand several fundamental aspects of vehicular traffic flow [1,2]. The ''particle-hopping'' models [3–5] of vehicular traffic are usually formulated using the language of cellular automata (CA) [6]. These models are closely related to some of the microscopic models of driven systems of interacting particles, which are of current interest in nonequilibrium statistical mechanics [7].

A one-dimensional CA model of highway traffic and a two-dimensional CA model of city traffic were developed independently by Nagel and Schreckenberg (NS) [3] and Biham, Middleton, and Levine (BML) [8], respectively. Highway traffic becomes gradually more and more congested in the NS model with the increase of density. Traffic jams appear in the NS model because of the *intrinsic stochasticity* of the dynamics [4] but no jam persists forever. On the other hand, a first order phase transition takes place in the BML model at a finite nonvanishing density, where the average velocity of the vehicles vanishes discontinuously, signaling complete jamming. In the BML model, the randomness, which is crucial for the jamming, arises only from the *random initial conditions*, as the dynamical rule for the movement of the vehicles is fully deterministic [8].

If each unit of discrete time interval in the BML model is interpreted as the time for which the traffic lights remain green (or red) before switching red (or green) simultaneously in a synchronized manner, then, over that time scale each vehicle, which faces a green signal, gets an opportunity to move from one crossing to the next. The generalization of the BML model that we propose here is an attempt to describe explicitly the forward movement of the vehicles over smaller distances during shorter time intervals. We achieve this generalization by following the prescriptions of the NS model not only for describing the positions, speeds, accelerations, and decelerations of the vehicles [9] but also for taking into account the interactions among the vehicles moving along the same lane of a street. We also modify some of the prescriptions of the BML model appropriately to take into account the signal-vehicle interactions and the interactions between vehicles approaching a crossing along different streets.

Our main aim is to demonstrate that a phase transition from the ''free-flowing'' dynamical phase to the completely ''jammed'' phase takes place in our generalized model; the intrinsic stochasticity of the dynamics, which triggers the onset of jamming, is similar to that in the NS model, while the phenomenon of complete jamming through self-organization as well as the final jammed configurations are similar to those in the BML model.

In the BML model a square lattice represents the network of the streets. All the streets parallel to the $\hat{X}$ direction of a Cartesian coordinate system are assumed to allow only single-lane east-bound traffic while all those parallel to the $\hat{Y}$ direction allow only single-lane north-bound traffic. Each of the lattice sites represents the crossing of a east-west street and a north-south street. In the initial state of the system, N_x (N_y) vehicles are distributed among the east-bound (north-bound) streets. The states of east-bound vehicles are updated in parallel at every odd discrete time step, whereas those of the north-bound vehicles are updated in parallel at every even discrete time step following a rule which is a simple extension of the fully asymmetric simple exclusion process [7]: a vehicle moves forward by one lattice spacing if and only if the site in front is empty, otherwise the vehicle does not move at that time step. Jamming arises from the mutual blocking of the flows of east-bound and north-bound traffic at various different crossings. The BML model has been modified and extended [10–17].

We model the network of the streets as a $N\times N$ square lattice. The streets parallel to X and Y axes allow only east-bound and north-bound traffic, respectively, as in the original formulation of the BML model. A signal is installed at every site of this $N\times N$ square lattice, where each of the sites represents a crossing of two mutually perpendicular streets. The separation between any two successive crossings on every

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street is assumed to consist of D cells so that the total number of cells on every street is $L=N\times D$. The linear size of each cell may be interpreted as the typical length of a car; each of these cells can be either empty or occupied by at most one single vehicle at a time. Because of these cells, the network of the streets can be viewed as a decorated lattice. However, unlike the BML model [8] and the model of Horiguchi and Sakakibara [14] which correspond to $D=1$ and $D=2$, respectively, $D(<L)$ in our model is to be treated as a parameter. Note that D introduces a new length scale into the problem.

The signals are synchronized in such a way that all the signals remain green for the east-bound vehicles (and simultaneously, red for the north-bound vehicles) for a time interval T and then, simultaneously, all the signals turn red for the east-bound vehicles (and, green for the north-bound vehicles). Thus, the parameter T introduces a new time scale into the problem.

As in the original BML model, no turning of the vehicles is allowed. Therefore, the total number of vehicles on each street is determined by the initial condition, and does not change with time because of the periodic boundary conditions.

Following the prescription of the NS model, we allow the speed V of each vehicle to take one of the $V_{max}+1$ *integer* values $V=0,1,\ldots,V_{max}$. Suppose V_n is the speed of the nth vehicle at time t while moving either towards east or towards north. At each *discrete time* step $t\rightarrow t+1$, the arrangement of N vehicles is updated *in parallel* according to the following ''rules'':

Step 1: Acceleration. If $V_n<V_{max}$, the speed of the nth vehicle is increased by one, i.e., $V_n\rightarrow V_n+1$.

Step 2: Deceleration (due to other vehicles or signal). Suppose d_n is the gap in between the nth vehicle and the vehicle in front of it, and s_n is the distance between the same nth vehicle and the closest signal in front of it.

Case I: The signal is **red** for the car under consideration:
If $\min(d_n,s_n)\leqslant V_n$, then $V_n\rightarrow\min(d_n,s_n)-1$.

Case II: The signal is **green** for the vehicle under consideration:
There are two possibilities in this case: (i) When $d_n<s_n$, then $V_n\rightarrow d_n-1$ if $d_n\leqslant V_n$, The motivation for this choice comes from the fact that, when $d_n<s_n$, the hindrance effect comes from the leading vehicle. (ii) When $d_n\geqslant s_n$, then $V_n\rightarrow\min(V_n,d_n-1)$ if $\min(V_n,d_n-1)\times\tau>s_n$, where τ is the number of the remaining time steps before the signal turns red. The motivation for this choice comes from the fact that, when $d_n\geqslant s_n$, the speed of the nth vehicle at the next time step depends on whether or not the vehicle can cross the crossing in front before the signal for it turns red.

Step 3: Randomization. If $V_n>0$, the speed of the nth vehicle is decreased randomly by unity (i.e., $V_n\rightarrow V_n-1$) with probability p $(0\leqslant p\leqslant 1)$; p, the random deceleration probability, is identical for all the vehicles and does not change during the updating.

Step 4: Vehicle movement. Each vehicle is moved forward so that for the east-bound vehicles, $X_n\rightarrow X_n+V_n$, where X_n denotes the position of the nth vehicle at time t while for the north-bound vehicles, $Y_n\rightarrow Y_n+V_n$, where Y_n denotes the position of the nth vehicle at time t.

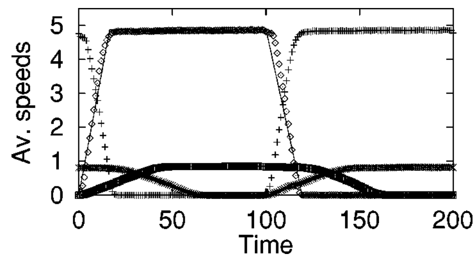

FIG. 1. Time dependence of average speeds of vehicles. The symbols $\diamondsuit$, $+$, $\square$, and $\times$ correspond, respectively, to $\langle V_x\rangle$ for $c=0.1$, $\langle V_y\rangle$ for $c=0.1$, $\langle V_x\rangle$ for $c=0.5$, and $\langle V_y\rangle$ for $c=0.5$. The common parameters are $V_{max}=5$, $p=0.1$, $D=100$, and $T=100$. The continuous line has been obtained from Eqs. (2) and (4).

These rules are not merely a combination of the rules proposed by BML [8] and those introduced by NS [3], but also involve some modifications. For example, unlike all the earlier BML-type models, a vehicle approaching a crossing can keep moving, even when the signal is red, until it reaches a site immediately in front of which there is either a halting vehicle or a crossing. Moreover, if $p=0$ every east-bound (north-bound) vehicle can adjust speed in the deceleration stage so as not to block the north-bound (east-bound) traffic when the signal is red for the east-bound (north-bound) vehicles.

In our computer simulations, we begin with an initial configuration where N_x and N_y vehicles are put at random positions on the east-bound and north-bound streets, respectively. The states of the vehicles are updated in parallel following the rules mentioned above. After the initial transients die down, at every time step, we compute the average speeds $\langle V_x\rangle$ and $\langle V_y\rangle$, which are merely the averages of the instantaneous speeds of the east-bound and north-bound vehicles, respectively. The density $c=(N_x+N_y)/(2LN-N^2)$ of the vehicles is the ratio of the total number of cars and the total number of cells in the system. Here we present the data for only the symmetric case $N_x=N_y$, for only a few sets of values of the parameters D,T,c,p,L,V_{max}; more details will be published elsewhere [18].

In the ''free-flowing'' phase of the BML model, both $\langle V_x\rangle$ and $\langle V_y\rangle$ oscillate between zero and a nonzero value periodically at odd and even time steps. In sharp contrast, the time dependences of $\langle V_x\rangle$ and $\langle V_y\rangle$ are much more realistic in our model, as is evident from Fig. 1. Moreover, as expected, V_g, the maximum allowed values of $\langle V_x\rangle$ and $\langle V_y\rangle$ in the corresponding green phase, is smaller when the density c is higher. In this parameter regime, following the switching of the red (green) signal to green (red), $\langle V_x\rangle$ rises (falls) to reach $V_g(0)$; the corresponding relaxation time is denoted by t_g (t_r). For a given c, we now derive approximate analytical expressions for t_g and t_r in terms of $V_{NS}(c)$, the steady speed of the vehicles, for the vehicle density c, in the NS model with periodic boundary conditions. Then, using the numerical estimates of $V_{NS}(c)$ from computer simulations of the NS model we compute $\langle V_x\rangle$ and $\langle V_y\rangle$ for our model and compare with the numerical data obtained from direct computer simulation.

We assume that during the red phase compact (i.e., without ''holes'') queues of length $N_q=cD$ are formed in front of each signal. We now estimate the time t_g until the stationary speed $V_g = V_{NS}(c)$ is reached. There are two different additive contributions to t_g. First, the last vehicle in a compact queue of N_q vehicles starts moving after $t_1 = N_q/(1-p)$ time steps, since the leading vehicle in the remainder of the queue moves with probability $1-p$. Second, a halting vehicle reaches the speed $V_{NS}(c)$ after a time $t_2 = V_{NS}(c)/(1-p)$ since it accelerates in each time step with probability $1-p$. Thus,

$$t_g = t_1 + t_2 = \frac{cD + V_{NS}(c)}{1-p}, \tag{1}$$

where we have assumed that the green phase starts at $t=0$. Moreover, under the assumption that the relaxation is perfectly linear we obtain the following average speeds during a green phase:

$$V_g(t) = \begin{cases} V_{NS}(c)t/t_g & \text{for } t < t_g, \\ V_{NS}(c) & \text{for } t \geqslant t_g \end{cases}. \tag{2}$$

In the stationary state of the green phase, there are, on the average, $1/c-1$ empty cells in front of each vehicle. So if the leading vehicle of a pair happens to be the last member of a queue already formed in front, then the following vehicle of that pair will move for $t_0 = (1/c-1)/V_{NS}(c)$ with velocity $V_{NS}(c)$ and then stop suddenly since it reaches the tail of a queue. Therefore, on the average, it takes a time

$$t_r = cDt_0 = \frac{(1-c)D}{V_{NS}(c)} \tag{3}$$

to form a queue of length $N_q = cD$ after the first vehicle has stopped at the red signal. In general, the vehicle nearest to a signal will not stop immediately after the signal turns red, but will keep moving for some time, say, t_p. The distance of this vehicle from the signal is expected to be a fraction α of the average distance $1/c-1$ to the next vehicle ahead of it. Taking $\alpha = 1/2$, for example, one obtains $t_p = 1 - c/2cV_{NS}(c)$. Hence, the average speed during a red phase starting at $t=0$ is given by

$$V_r(t) = \begin{cases} V_{NS}(c) & \text{for } 0 < t < t_p, \\ V_{NS}(c)\left[1 - \dfrac{t-t_p}{t_r}\right] & \text{for } t_p < t < t_p + t_r, \\ 0 & \text{for } t \geqslant t_p + t_r. \end{cases} \tag{4}$$

For the validity of these estimates $T(\gg 1)$ should be sufficiently large to guarantee complete queueing of the vehicles during the red phase, p should be small enough to ensure compactness of the queues, c should be sufficiently small so that the vehicles emerging from a queue should not be hindered by the halting vehicles of another queue in front. Moreover, the smaller is V_{max}, the stronger is the deviation from the linear relaxation assumed above. Furthermore, we have assumed that all queues have the same length.

In Fig. 1, $T=100$ is sufficiently long so that $\langle V_x \rangle$ and $\langle V_y \rangle$ relax to $V_{NS}(c)$ during the green phase of the corresponding signals and to zero during the red phase of the signals. In contrast, for the same values of the parameters D and c, the average speeds $\langle V_x \rangle$ and $\langle V_y \rangle$ do not get sufficient

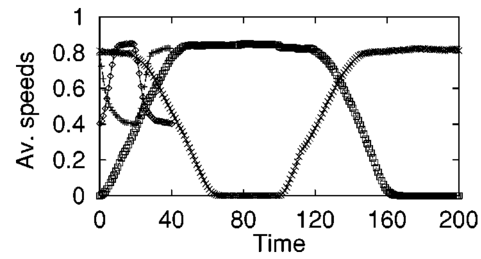

FIG. 2. Time dependence of average speeds of vehicles. The symbols $\diamond$, $+$, $\square$, and $\times$ correspond, respectively, to $\langle V_x \rangle$ for $T=20$, $\langle V_y \rangle$ for $T=20$, $\langle V_x \rangle$ for $T=100$, and $\langle V_y \rangle$ for $T=100$. The common parameters are $V_{max}=5$, $p=0.1$, $D=100$, and $c=0.5$. The data for $T=20$ are shown only up to 40 time steps to avoid overcrowding of data points. The lines are to serve as guides to the eye.

time to relax to zero during the red phase of the corresponding signal provided T is sufficiently small, e.g., $T=20$ (see Fig. 2).

The most dramatic result of our investigation is that at a sufficiently large density $c_*(D,T)$, which depends on D and T, a phase transition from the ''free-flowing'' dynamical phase to a completely jammed phase can take place in our ''unified'' model. In the jammed phase, the flow of east-bound vehicles is blocked by the north-bound vehicles and vice versa; this gridlock phenomenon, as well as the typical configurations in the jammed phase (see Fig. 3) of our ''unified'' model are similar to those in the BML model. In spite of these apparent similarities, as we shall explain now, the mechanism that triggers jamming in our ''unified'' model is different from that in the BML model. It is obvious from the updating rules that if $p=0$, i.e., if no random braking takes place, complete jamming is impossible in this model at any density $c<1$. Therefore, a vehicle that is located at the crossing of two mutually perpendicular streets and whose instantaneous speed is $V=1$ at the end of the deceleration stage

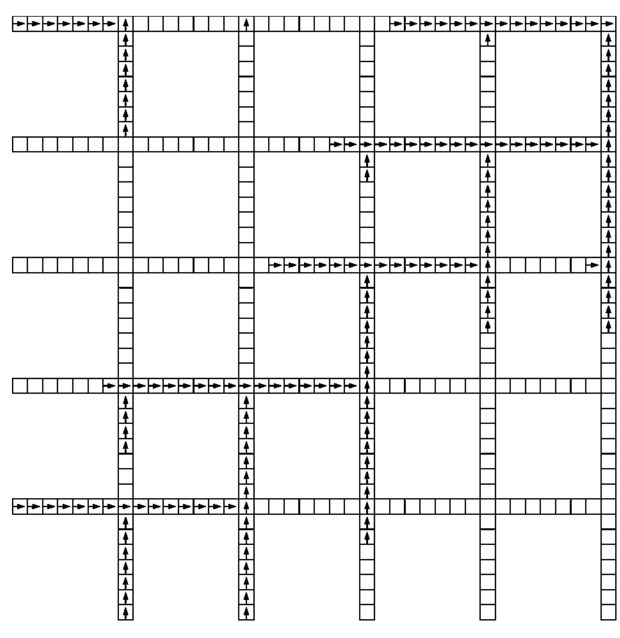

FIG. 3. Typical jammed configuration of the vehicles. The east-bound and north-bound vehicles are represented by the symbols $\rightarrow$ and $\uparrow$, respectively.

(i.e., having at least one empty site in front of it) would vacate the crossing unless its speed is reduced to $V=0$ because of random braking. However, if such a halt at a crossing due to random braking takes place at the last time step before the signal for it turns red, it would not only continue to block the perpendicular flow of traffic through the same crossing during the next T time steps, but would also give rise to a queue of jammed vehicles in the perpendicular street passing through the same crossing. Therefore, when $p\neq 0$ and the density is sufficiently high, the dynamical phase of ''freely-flowing'' traffic becomes unstable against the spontaneous formation of jams and the entire traffic system self-organizes so as to reach the completely jammed state.

Moreover, for given D, the shorter is the time interval T the smaller is the c_* (see Fig. 4). Besides, the density corresponding to the maximum flux also shifts to smaller densities with the decrease of T. Furthermore, the maximum throughput is a nonmonotonic function of T in the ''free-flowing'' phase; this result may be of practical use in traffic engineering for maximizing the throughput.

In this Rapid Communication we have developed a ''unified'' model where the jams are created by the same stochastic process as in the NS model, but the transition to complete jamming and the jammed configurations are very similar to those in the BML model. We have also established that our model describes the time dependence of the average speeds of the vehicles in a more realistic manner than any of the earlier CA models of the BML type. The results of our ongoing investigations of the effects of (a) turning of vehicles from east-bound (north-bound) streets to north-bound (east-bound) streets and (b) green-wave signaling on the flow and jamming will be reported elsewhere [18].

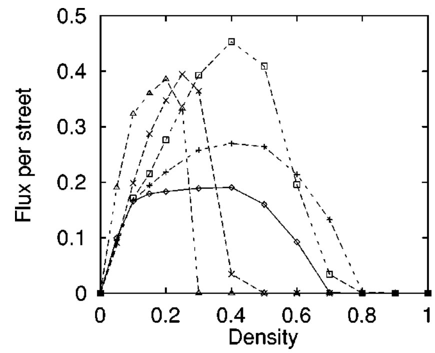

FIG. 4. Fundamental diagram. The symbols $\diamond$, $+$, $\Box$, $\times$, and $\triangle$ correspond, respectively, to $T=100$, 50, 20, 10, and 4. The common parameters are $V_{max}=5$, $p=0.5$, $L=100$, and $D=20$.

We thank Ludger Santen for useful discussions as well as for help in producing Fig. 3, and Dietrich Stauffer for valuable comments on the manuscript. This work is supported in part by the SFB341 Aachen-Jülich-Köln.

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